Puzzle of four sons and paddy bags

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Pinaki Chakrabarti asked this puzzle in the *MCC June 2002 puzzle contest*.

1 Question

A farmer had 4 sons $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$. One day the farmer asked them to go to the paddy field and to get the paddy in the bags. All four obeyed the order and after returning from the field each of them gave bags (full of paddy) to their father. It should be noted that they returned at different times and thus none of them was aware of the number of the bags that others had given to their father. Now the farmer called all his sons in the evening and told them that he would give them five clues and they have to tell the number of the bags each of them had brought. The five clues are here :

(i) There were 22 bags.

(ii) Each of them brought different number of bags.

(iii) $\mathcal{A}$ brought the maximum number of bags.

(iv) Number of bags filled by $\mathcal{A}$ is equal to the sum of the number of bags filled by $\mathcal{B}$ and $\mathcal{C}$.

(v) Each son brought at least one bag.*

Now, the farmer asked $\mathcal{A}$. But he couldn't answer. Same thing happened for $\mathcal{B}$ and $\mathcal{C}$ also, in order. At last when he asked $\mathcal{D}$, he answered properly.

Can you tell me what is the answer that $\mathcal{D}$ told to his father ? As an additional clue, I can say that the number of the bags that $\mathcal{B}$ has filled is not the minimum of the four.

*This clue was not in the original puzzle statement. This is my addition. See §4 on page 4 for a discussion.

2 Answer

$\mathcal{A}$ brought 9, $\mathcal{B}$ brought 6, $\mathcal{C}$ brought 3, $\mathcal{D}$ brought 4.

3 Solution

This puzzle requires a combination of arithmetic and logic.

3.1 Arithmetic part:

Let the sons brought a, b, c and d number of bags. We can assume that each of these numbers is greater than zero and less than 23.

Now, since $a = b + c$, we can say $(2a + d) = 22$, so a and d are related. Since condition (iii) states a is the largest of the four, a should be at least 8. ($a = 7$ will make $d = 8$). Also, a should be at most 10, because $a = 11$ will make $d = 0$.

So, the value of a is one of 8, 9 or 10. And the value of d is 6, 4 or 2 respectively. Also, the four values should be different. The possible combinations are:

```
( C1) ( 8, 7, 1, 6)
( C2) ( 8, 5, 3, 6)
( C3) ( 8, 3, 5, 6)
( C4) ( 8, 1, 7, 6)
( C5) ( 9, 8, 1, 4)
( C6) ( 9, 7, 2, 4)
( C7) ( 9, 6, 3, 4)
( C8) ( 9, 3, 6, 4)
( C9) ( 9, 2, 7, 4)
(C10) ( 9, 1, 8, 4)
(C11) (10, 9, 1, 2)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
(C14) (10, 4, 6, 2)
(C15) (10, 3, 7, 2)
(C16) (10, 1, 9, 2)
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3.2 logic part:

$\mathcal{A}$ and $\mathcal{D}$ knows their own and the other's value. They need to know at least one of b or c to solve the puzzle. $\mathcal{B}$ and $\mathcal{C}$ knows only their own value. They need to know at least one more to solve the puzzle.

$\mathcal{A}$ has 8 in 4 cases, 9 in 6 cases, and 10 in 6 cases. So, $\mathcal{A}$ has no clue whatever his value may be.

$\mathcal{B}$ has 1 in 3 cases, 2 in one case, 3 in 3 cases, 4 in one case, 5 in one case, 6 in 2 cases, 7 in 3 cases, 8 in one case and 9 in one case.

Since $\mathcal{B}$ could not get the values, his value does not belong to a unique case. In other words, his value is not any of 2, 4, 5, 8 and 9. So, the following cases are not possible.

C9, C14, C2, C5, C11.

```
( C1) ( 8, 7, 1, 6)
( C2) ( 8, 5, 3, 6)   [struck through]
( C3) ( 8, 3, 5, 6)
( C4) ( 8, 1, 7, 6)
( C5) ( 9, 8, 1, 4)   [struck through]
( C6) ( 9, 7, 2, 4)
( C7) ( 9, 6, 3, 4)
( C8) ( 9, 3, 6, 4)
( C9) ( 9, 2, 7, 4)   [struck through]
(C10) ( 9, 1, 8, 4)
(C11) (10, 9, 1, 2)   [struck through]
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
(C14) (10, 4, 6, 2)   [struck through]
(C15) (10, 3, 7, 2)
(C16) (10, 1, 9, 2)
```

So, when the problem is presented to C, only 11 cases are possible:

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( C1) ( 8, 7, 1, 6)
( C3) ( 8, 3, 5, 6)
( C4) ( 8, 1, 7, 6)
( C6) ( 9, 7, 2, 4)
( C7) ( 9, 6, 3, 4)
( C8) ( 9, 3, 6, 4)
(C10) ( 9, 1, 8, 4)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
(C15) (10, 3, 7, 2)
(C16) (10, 1, 9, 2)
```

Here $\mathcal{C}$ has 1 in one case, 2 in one case, 3 in 2 cases, 4 in one case, 5 in one case, 6 in one case, 7 in two cases, 8 in one case, and 9 in one case. All one-case cases can be eliminated, because $\mathcal{C}$ could not answer. So, the remaining cases are:

~~(C1) (8, 7, 1, 6)~~
~~(C3) (8, 3, 5, 6)~~
(C4) (8, 1, 7, 6)
~~(C6) (9, 7, 2, 4)~~
(C7) (9, 6, 3, 4)
~~(C8) (9, 3, 6, 4)~~
~~(C10) (9, 1, 8, 4)~~
(C12) (10, 7, 3, 2)
~~(C13) (10, 6, 4, 2)~~
(C15) (10, 3, 7, 2)
~~(C16) (10, 1, 9, 2)~~

Which means the following four cases:

```
( C4) ( 8, 1, 7, 6)
( C7) ( 9, 6, 3, 4)
(C12) (10, 7, 3, 2)
(C15) (10, 3, 7, 2)
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Now, $\mathcal{D}$ *could* answer. That means $d \neq 2$, because if d were 2, he could not have identified which of the two cases (C12) or (C15).

Now, the case must be either (C4) or (C7). $\mathcal{D}$ doesn't have confusion, because he knows his value, but we do. The final clue (given to us, not to them) that $\mathcal{B}$ doesn't have the minimum value eliminates (C4). So, the answer is (C7).

So,

- $\mathcal{A}$ brought 9,
- $\mathcal{B}$ brought 6,
- $\mathcal{C}$ brought 3, and
- $\mathcal{D}$ brought 4.

4 Tries, traps and pitfalls

1. *Why is the assumption that each son brought at least one bag is important?*

   We know from the final answer that everyone brought at least one bag. However, it is important that it is explicitely stated *and* everyone knew it; otherwise, the puzzle is insolvable. Let us see what happens if one of them *could* bring zero bags.

   It is clear that only $\mathcal{D}$ can have zero, (otherwise, other conditions will be violated), but it will introduce the following cases:

(C17) (11, 1, 10, 0)
(C18) (11, 2, 9, 0)
(C19) (11, 3, 8, 0)
(C20)(11, 4, 7, 0)
(C21) (11, 5, 6, 0)
(C22) (11, 6, 5, 0)
(C23) (11, 7, 4, 0)
(C24) (11, 8, 3, 0)
(C25) (11, 9, 2, 0)
(C26) (11, 10, 1, 0)

These cases represent all values for $\mathcal{B}$ and C. As a result, no case (in the original set) can be eliminated by A, $\mathcal{B}$ or C, and the entire set is presented to D. $\mathcal{D}$ will be completely clueless to answer.

However, one can argue: "Hey, the puzzle states that each of them gave *bags* to their father, so we can assume that each gave at least one, and everyone knows that". Right, but in that case, each should have brought at least *two* (because of *bags*, which is plural), thus eliminating (C1), (C4), (C5), (C10), (C11) and (C16) also in the beginning. Let us try again with it:

$\mathcal{B}$ gets the following, and eliminates its unique values:

~~(C2) (8, 5, 3, 6)~~
(C3) (8, 3, 5, 6)
(C6) (9, 7, 2, 4)
(C7) (9, 6, 3, 4)
(C8) (9, 3, 6, 4)
~~(C9) (9, 2, 7, 4)~~
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
~~(C14) (10, 4, 6, 2)~~
(C15) (10, 3, 7, 2)

Now, $\mathcal{C}$ eliminates his own unique values:

~~(C3) (8, 3, 5, 6)~~
~~(C6) (9, 7, 2, 4)~~
(C7) (9, 6, 3, 4)
~~(C8) (9, 3, 6, 4)~~
(C12) (10, 7, 3, 2)
~~(C13) (10, 6, 4, 2)~~
~~(C15) (10, 3, 7, 2)~~

leaving this to $\mathcal{D}$.

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( C7) ( 9, 6, 3, 4)
(C12) (10, 7, 3, 2)
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$\mathcal{D}$ could answer this, but we cannot. So, this is indeterminate.

So, depending on the grammar is not right. The question is what does "each of them gave *bags* to their father" mean?

Each of them brought at least one. This is assumed to solve the puzzle. And, it is assumed that *they* knew this.

Each of them brought at least two. This is fine, for the final answer. But it is not clear whether they knew this. If they did, this puzzle is insolvable.

Nothing about the number each brought. Can be anything from 0 to 22, subject to other conditions. But, allowing 0 makes the problem insolvable.

That is why that assumption is explicitely stated.

4.1 What is the most dangerous pitfall in this puzzle?

Assuming that the final clue (that $\mathcal{B}$ didn't bring the minimum number of bags) is known to everybody. This will eliminate more cases too early, making the puzzle insolvable. See below:

After the arithmetic analysis, we get the following combinations:

```
( C1) ( 8, 7, 1, 6)
( C2) ( 8, 5, 3, 6)
( C3) ( 8, 3, 5, 6)
( C4) ( 8, 1, 7, 6)
( C5) ( 9, 8, 1, 4)
( C6) ( 9, 7, 2, 4)
( C7) ( 9, 6, 3, 4)
( C8) ( 9, 3, 6, 4)
( C9) ( 9, 2, 7, 4)
(C10) ( 9, 1, 8, 4)
(C11) (10, 9, 1, 2)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
(C14) (10, 4, 6, 2)
(C15) (10, 3, 7, 2)
(C16) (10, 1, 9, 2)
```

If they knew that $\mathcal{B}$ didn't bring the least amount, they can eliminate a few cases:

(C1) (8, 7, 1, 6)
(C2) (8, 5, 3, 6)
~~(C3) (8, 3, 5, 6)~~
~~(C4) (8, 1, 7, 6)~~
(C5) (9, 8, 1, 4)
(C6) (9, 7, 2, 4)
(C7) (9, 6, 3, 4)
~~(C8) (9, 3, 6, 4)~~
~~(C9) (9, 2, 7, 4)~~
~~(C10) (9, 1, 8, 4)~~
(C11) (10, 9, 1, 2)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
(C14) (10, 4, 6, 2)
(C15) (10, 3, 7, 2)
~~(C16) (10, 1, 9, 2)~~

$\mathcal{A}$ is still clueless, but $\mathcal{B}$ will get the following:

(C1) (8, 7, 1, 6)
(C2) (8, 5, 3, 6)
(C5) (9, 8, 1, 4)
(C6) (9, 7, 2, 4)
(C7) (9, 6, 3, 4)
(C11) (10, 9, 1, 2)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
(C14) (10, 4, 6, 2)
(C15) (10, 3, 7, 2)

Now removing cases where $\mathcal{B}$ has only one case:

(C1) (8, 7, 1, 6)
~~(C2) (8, 5, 3, 6)~~
~~(C5) (9, 8, 1, 4)~~
(C6) (9, 7, 2, 4)
(C7) (9, 6, 3, 4)
~~(C11) (10, 9, 1, 2)~~
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
~~(C14) (10, 4, 6, 2)~~
~~(C15) (10, 3, 7, 2)~~

This is what is presented to C.

```
( C1) ( 8, 7, 1, 6)
( C6) ( 9, 7, 2, 4)
( C7) ( 9, 6, 3, 4)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
```

Eliminating the cases where $\mathcal{C}$ has only one case, it becomes:

~~(C1) (8, 7, 1, 6)~~
~~(C6) (9, 7, 2, 4)~~
`( C7) ( 9, 6, 3, 4)`
`(C12) (10, 7, 3, 2)`
~~(C13) (10, 6, 4, 2)~~

$\mathcal{D}$ *could* answer, because he knows how many he has. But *we* do not know that. We are stuck here.

Many people proceeded this way, and after getting these two values, they interpreted that the meaning of *As an additional clue, I can say that the number of the bags that $\mathcal{B}$ has filled is not the minimum* in the puzzle as *you will get two answers, and discard the one where $\mathcal{B}$ has the minimum of the two.*, and gave (10, 7, 3, 2) as the answer. That way, it can be considered a *try*.